

90 нужных формул тригонометрии

$$1. \operatorname{tg} \alpha = \frac{\sin \alpha}{\cos \alpha}$$

$$2. \operatorname{ctg} \alpha = \frac{\cos \alpha}{\sin \alpha}$$

$$3. \cos^2 \alpha + \sin^2 \alpha = 1$$

$$4. \operatorname{tg} \alpha = \frac{1}{\operatorname{ctg} \alpha}$$

$$5. \operatorname{ctg} \alpha = \frac{1}{\operatorname{tg} \alpha}$$

$$6. \operatorname{tg} \alpha \cdot \operatorname{ctg} \alpha = 1$$

$$7. 1 + \operatorname{ctg}^2 \alpha = \frac{1}{\sin^2 \alpha}$$

$$8. 1 + \operatorname{tg}^2 \alpha = \frac{1}{\cos^2 \alpha}$$

$$9, 10. \sin(\alpha \pm \beta) = \sin \alpha \cdot \cos \beta \pm \cos \alpha \cdot \sin \beta$$

$$11, 12. \cos(\alpha \pm \beta) = \cos \alpha \cdot \cos \beta \mp \sin \alpha \cdot \sin \beta$$

$$13, 14. \operatorname{ctg}(\alpha \pm \beta) = \frac{\operatorname{ctg} \alpha \cdot \operatorname{ctg} \beta \mp 1}{\operatorname{ctg} \beta \pm \operatorname{ctg} \alpha}$$

$$15, 16. \operatorname{tg}(\alpha \pm \beta) = \frac{\operatorname{tg} \alpha \pm \operatorname{tg} \beta}{1 \mp \operatorname{tg} \alpha \cdot \operatorname{tg} \beta}$$

$$17. \sin 2\alpha = 2 \sin \alpha \cdot \cos \alpha$$

$$18. \cos 2\alpha = 1 - 2 \sin^2 \alpha$$

$$19. \cos 2\alpha = 2 \cos^2 \alpha - 1$$

$$20. \cos 2\alpha = \cos^2 \alpha - \sin^2 \alpha$$

$$21. \sin 3\alpha = 3 \sin \alpha - 4 \sin^3 \alpha$$

$$22. \sin 4\alpha = 8 \cos^3 \alpha \cdot \sin \alpha - 4 \cos \alpha \cdot \sin \alpha$$

$$23. \cos 3\alpha = 4 \cos^3 \alpha - 3 \cos \alpha$$

$$24. \cos 4\alpha = 8 \cos^4 \alpha - 8 \cos^2 \alpha + 1$$

$$25. \operatorname{tg} 2\alpha = \frac{2 \operatorname{tg} \alpha}{1 - \operatorname{tg}^2 \alpha}$$

$$26. \operatorname{ctg} 2\alpha = \frac{\operatorname{ctg} \alpha - 1}{2 \operatorname{ctg} \alpha}$$

$$27. \operatorname{tg} 3\alpha = \frac{3 \operatorname{tg} \alpha - \operatorname{tg}^3 \alpha}{1 - 3 \operatorname{tg}^2 \alpha}$$

$$28. \operatorname{ctg} 3\alpha = \frac{\operatorname{ctg}^3 \alpha - 3 \operatorname{ctg} \alpha}{3 \operatorname{ctg}^2 \alpha - 1}$$

$$29. \operatorname{tg} 4\alpha = \frac{4 \operatorname{tg} \alpha - 4 \operatorname{tg}^3 \alpha}{1 - 6 \operatorname{tg}^2 \alpha + \operatorname{tg}^4 \alpha}$$

$$30. \operatorname{ctg} 4\alpha = \frac{\operatorname{ctg}^4 \alpha - 6\operatorname{ctg}^2 \alpha + 1}{4\operatorname{ctg}^3 \alpha - 4\operatorname{ctg} \alpha}$$

$$31. \sin \frac{\alpha}{2} = \sqrt{\frac{1 - \cos \alpha}{2}}$$

$$32. \cos \frac{\alpha}{2} = \sqrt{\frac{1 + \cos \alpha}{2}}$$

$$33, 34. \operatorname{tg} \frac{\alpha}{2} = \frac{\sin \alpha}{1 + \cos \alpha} = \frac{1 - \cos \alpha}{\sin \alpha}$$

$$35, 36. \operatorname{ctg} \frac{\alpha}{2} = \frac{\sin \alpha}{1 - \cos \alpha} = \frac{1 + \cos \alpha}{\sin \alpha}$$

$$37. \sin \alpha + \sin \beta = 2 \sin \frac{\alpha + \beta}{2} \cdot \cos \frac{\alpha - \beta}{2}$$

$$38. \sin \alpha - \sin \beta = 2 \sin \frac{\alpha - \beta}{2} \cdot \cos \frac{\alpha + \beta}{2}$$

$$39. \cos \alpha + \cos \beta = 2 \cos \frac{\alpha + \beta}{2} \cdot \cos \frac{\alpha - \beta}{2}$$

$$40. \cos \alpha - \cos \beta = -2 \sin \frac{\alpha + \beta}{2} \cdot \sin \frac{\alpha - \beta}{2}$$

$$41. \operatorname{tg} \alpha + \operatorname{tg} \beta = \frac{\sin(\alpha + \beta)}{\cos \alpha \cdot \cos \beta}$$

$$42. \operatorname{tg} \alpha - \operatorname{tg} \beta = \frac{\sin(\alpha - \beta)}{\cos \alpha \cdot \cos \beta}$$

$$43. \operatorname{ctg} \alpha + \operatorname{ctg} \beta = \frac{\sin(\alpha + \beta)}{\sin \alpha \cdot \sin \beta}$$

$$44. \operatorname{ctg} \alpha - \operatorname{ctg} \beta = \frac{-\sin(\alpha - \beta)}{\sin \alpha \cdot \sin \beta}$$

$$45. \cos \alpha + \sin \alpha = \sqrt{2} \cdot \cos(45^\circ - \alpha)$$

$$46. \cos \alpha - \sin \alpha = \sqrt{2} \cdot \sin(45^\circ - \alpha)$$

$$47. \operatorname{tg} \alpha + \operatorname{ctg} \beta = \frac{\cos(\alpha - \beta)}{\cos \alpha \cdot \sin \beta}$$

$$48. \operatorname{tg} \alpha - \operatorname{ctg} \beta = \frac{-\cos(\alpha + \beta)}{\cos \alpha \cdot \sin \beta}$$

$$49. \operatorname{tg} \alpha - \operatorname{ctg} \alpha = -2 \operatorname{tg} 2\alpha$$

$$50. 1 + \cos \alpha = 2 \cos^2 \frac{\alpha}{2}$$

$$51. 1 - \cos \alpha = 2 \sin^2 \frac{\alpha}{2}$$

$$52. 1 + \sin \alpha = 2 \cos^2 \left(45^\circ - \frac{\alpha}{2}\right)$$

$$53. 1 - \sin \alpha = 2 \sin^2 \left(45^\circ - \frac{\alpha}{2} \right)$$

$$54. \sin^2 \alpha = \frac{1 - \cos 2\alpha}{2}$$

$$55. \sin^3 \alpha = \frac{1}{4} (3 \sin \alpha - \sin 3\alpha)$$

$$56. \cos^2 \alpha = \frac{1 + \cos 2\alpha}{2}$$

$$57. \cos^3 \alpha = \frac{1}{4} (\cos 3\alpha + 3 \cos \alpha)$$

$$58. \sin^4 \alpha = \frac{1}{8} (\cos 4\alpha - 4 \cos 2\alpha + 3)$$

$$59. \cos^4 \alpha = \frac{1}{8} (\cos 4\alpha + 4 \cos 2\alpha + 3)$$

60.

$$\sin \alpha \cdot \cos \beta = \frac{1}{2} (\sin(\alpha + \beta) + \sin(\alpha - \beta))$$

$$61. \cos \alpha \cdot \cos \beta = \frac{1}{2} (\cos(\alpha + \beta) + \cos(\alpha - \beta))$$

$$62. \sin \alpha \cdot \sin \beta = \frac{1}{2} (\cos(\alpha - \beta) - \cos(\alpha + \beta))$$

$$63. \arcsin x = -\arcsin(-x) = \frac{\pi}{2} - \arccos x = \operatorname{arctg} \frac{x}{\sqrt{1-x^2}}$$

$$64. \arccos x = \pi - \arccos(-x) = \frac{\pi}{2} - \arcsin x = \operatorname{arctg} \frac{x}{\sqrt{1-x^2}}$$

$$65. \operatorname{arctg} x = -\operatorname{arctg}(-x) = \frac{\pi}{2} - \operatorname{arctg} x = \arcsin \frac{x}{\sqrt{1+x^2}}$$

$$66. \operatorname{arctg} x = \pi - \operatorname{arctg}(-x) = \frac{\pi}{2} - \operatorname{arctg} x = \arccos \frac{x}{\sqrt{1+x^2}}$$

$$67. \arcsin(-x) = -\arcsin x$$

$$68. \arccos(-x) = \pi - \arccos x$$

$$69. \operatorname{arctg}(-x) = -\operatorname{arctg} x$$

$$70. \operatorname{arctg}(-x) = \pi - \operatorname{arctg} x$$

$$71. \cos \alpha = \frac{1 - \operatorname{tg}^2 \frac{\alpha}{2}}{1 + \operatorname{tg}^2 \frac{\alpha}{2}}$$

$$72. \sin \alpha = \frac{2 \operatorname{tg} \frac{\alpha}{2}}{1 + \operatorname{tg}^2 \frac{\alpha}{2}}$$

$$73. \begin{aligned} \sin x &= a; |a| < 1 \\ x &= (-1)^n \arcsin a + \pi n, n \in Z \end{aligned}$$

$$74. \begin{aligned} \cos x &= a; |a| < 1 \\ x &= \pm \arccos a + 2\pi n, n \in Z \end{aligned}$$

$$75. \begin{aligned} \operatorname{tg} x &= a \\ x &= \operatorname{arctg} a + \pi n, n \in Z \end{aligned}$$

$$76. \begin{aligned} \operatorname{ctg} x &= a \\ x &= \operatorname{arctg} a + \pi n, n \in Z \end{aligned}$$

$$77. \begin{aligned} \text{при } |a| \leq 1 \\ 0 \leq \arccos a \leq \pi \\ \cos(\arccos a) &= a \end{aligned}$$

$$78. \begin{aligned} \text{при } |a| \leq 1 \\ -\frac{\pi}{2} \leq \arcsin a \leq \frac{\pi}{2} \\ \sin(\arcsin a) &= a \end{aligned}$$

$$79. \begin{aligned} \text{при } \forall a \\ -\frac{\pi}{2} < \operatorname{arctg} a < \frac{\pi}{2} \\ \operatorname{tg}(\operatorname{arctg} a) &= a \end{aligned}$$

$$80. \begin{aligned} \text{при } \forall a \\ 0 < \operatorname{arctg} a < \pi \\ \operatorname{ctg}(\operatorname{arctg} a) &= a \end{aligned}$$

$$81. \arcsin a + \arccos a = \frac{\pi}{2}$$

$$82. \operatorname{arctg} a + \operatorname{arctg} a = \frac{\pi}{2}$$

$$83. \begin{aligned} \sin x &= 0 \\ x &= \pi n, n \in Z \end{aligned}$$

$$\cos x = 0$$

$$84. \quad x = \frac{\pi}{2} + \pi n, n \in Z$$

$$\sin x = 1$$

$$85. \quad x = \frac{\pi}{2} + 2\pi n, n \in Z$$

$$\cos x = 1$$

$$86. \quad x = 2\pi n, n \in Z$$

$$\arcsin x = a$$

$$87. \quad -\frac{\pi}{2} \leq a \leq \frac{\pi}{2}$$

$$x = \sin a$$

$$\arccos x = a$$

$$88. \quad 0 \leq a \leq \pi$$

$$x = \cos a$$

$$\operatorname{arctg} x = a$$

$$89. \quad -\frac{\pi}{2} < a < \frac{\pi}{2}$$

$$x = \operatorname{tga}$$

$$\operatorname{arccotg} x = a$$

$$90. \quad 0 \leq a \leq \pi$$

$$x = \operatorname{ctga}$$